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On Red Shifts and Asymmetries of Hydrogen Spectral Lines

B. Grabowski and J. Halenka
Institute of Physics, Higher Pedagogical School, Opole, Poland

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Summary. In the present paper, the internal electric field in hydrogen atoms, perturbed by intermolecular interactions in a plasma, is described by the Debye model. Using this model, we have determined the energy structure of low-lying levels. The structure obtained is deformed—with respect to the unperturbed structure—in such a way that the spectral lines are shifted towards the red. In so far as the order of magnitude and the direction are concerned, the predicted shifts agree with the recent detailed observations of high-density laboratory plasma. It would appear that the observed asymmetry of hydrogen lines can be also explained with this model.

Key words: red shifts — asymmetry of hydrogen lines

1. Introduction

The half-widths and profiles of hydrogen lines, broadened by the intermolecular Stark effect, are sensitive indicators of electron density, being almost independent of temperature. Therefore, an exact theoretical description of the broadening of hydrogen lines is important from the point of view of astronomical and laboratory spectroscopy.

The theory of Stark broadening of hydrogen lines in plasma, developed during the last fifteen years, predicts in principle symmetric and unshifted profiles. [For a comment on "trivial" sources of asymmetries and shifts of Stark-broadened hydrogen and hydrogenic lines see e.g. Griem (1964, Section 4–11).] This theory—especially after recent improvements by Kepple and Griem (1968) (also by Smith *et al.*, 1968, 1971)—applied to the first lines of the Lyman and Balmer series agrees with observations to within 10% or better. However, the latest detailed measurements of the line profiles in high-density plasmas, made by Wiese and Kelleher (1971), Wiese (1972) and Wiese *et al.* (1972), point out appreciable red shifts and asymmetries of hydrogen lines, which are not predicted by any current broadening theory.

The former effect is of special astrophysical interest in connection with the relativistic red shifts of the hydrogen Balmer lines observed in white-dwarf spectra. The measurements referred to above have shown that a certain broadening effect, which appears in dense plasmas only, accounts for a considerable part of the astrophysically observed residual red shifts, which have previously been attributed entirely to gravitation (cf. Wiese and Kelleher, 1971).

The present paper is the result of an attempt to explain theoretically the red shifts and asymmetries of the hydrogen lines observed in laboratory plasmas. We have shown that both these features, at least in so far as the direction and the order of the magnitude are concerned, can be explained as a result of the screening effect.

2. Observed Features

An asymmetry between the blue and red peaks of H_β line has often been observed (e.g. Griem, 1954). However, hydrogen and hydrogenic lines have been implicitly assumed to be symmetrical. The papers of Wiese and his co-workers are probably the most accurate and comprehensive experimental studies of the hydrogen-line profiles, and they show that in dense plasmas the asymmetries of the H_β as well as of the H_γ lines are systematic and very reproducible features, extending from the centers out to the far wings. (Within the precision of the measurements, H_α does not exhibit asymmetry; the overlapping wings of the subsequent Balmer lines prevent the measurement of asymmetries of lines further than H_γ.)

The main conclusions of Wiese *et al.* are as follows:

(a) The asymmetries in H_β and H_γ always enhance the blue side over the red side near the line peak; however, away from the line center a crossover occurs, and the red wing is enhanced over the blue one.

(b) The red shifts, somewhat smaller for H_α and very reproducible for H_β and H_γ, are approximately linear functions of electron density.

(c) The major discrepancy between the calculations and experiment occurs at the line centers, viz. the experiments show systematically less structure than recent theoretical profiles predict (cf. also Kelleher and Wiese, 1973).

Further effects are suggested by Bengston and Chester (1972). Unfortunately, their measurements of the late members of the Balmer series (H_{13}–H_{17}), emitted from a low-density and low-temperature laboratory plasma, are much less accurate. Therefore, we note only the more reliable results:

(d) Only Balmer lines originating from odd upper levels display measurable red shifts[1]).

(e) The shift increases with the upper principal quantum number of the line.

(f) The measured shifts of H_{15} and H_{17} are relatively big: of the order of 10% and 20% of the Stark (full) half-widths, respectively.

We note that all the red-shift measurements of Wiese *et al.* were performed for the so-called "experimental line center", defined as the symmetry axis of the three points that bisect the 1/2, 1/4 and 1/8 widths. The line center defined in this way was first used to fit laboratory measurements to astrophysical ones in white-dwarf spectra (Wiese and Kelleher, 1971), and, in the case of asymmetric lines, does not coincide with either the line peak or its center of gravity. The observed magnitudes of the red shifts, defined as above, range from nearly zero to 0.5, 0.6 and 0.8 Å for H_α, H_β and H_γ, respectively, as the electron density changes from $10^{16}\,cm^{-3}$ to $9\times10^{16}\,cm^{-3}$ at an approximately constant temperature of about 10000 K.

It is clear that the corresponding red shifts of the centers of gravity of H_β and H_γ lines should be very much larger. This should probably apply also to the measurements made by Bengston and Chester, where the line center was defined in a still narrower region, ranging from only 3/4 to 1/4 of the maximum intensity of the line. Unfortunately, it is extremely difficult to measure the red shifts of the centers of gravity of lines, and we know of no published data on this subject.

In the paper of Wiese and Kelleher (1971, Fig. 1) it is shown clearly that the measured red shift depends critically on the definition of the line center: one obtains a smaller shift when the 1/8-width, representing the nearby line-wings, is omitted from this definition. More outlying wings, of course, affect still more critically the result obtained. We shall discuss the effect on the most important, and consequently, the most studied line-H_β (cf. Wiese *et al.*, 1972, p. 1147). Complete profile measurements in the absolute wavelength scale are only available for this line (see Wiese, 1972, Fig. 6).

For H_β under the plasma conditions $N_e=8.0\times10^{16}\,cm^{-3}$, $T=12700$ K we obtained the following red shifts of the symmetry axes: 0.4, 0.5 and 0.9 Å at 1/2, 1/4 and 1/8 of the peak height, respectively. These numbers agree with the mean red shift of about 0.6 Å obtained by Wiese (1972, Fig. 3) and Wiese *et al.* (1972, Fig. 7) for H_β for very similar physical conditions ($N_e=8.3\times10^{16}\,cm^{-3}$, $T=13400$ K). However, when the symmetry axes are taken at the widths bisected at progressively smaller heights, 1/10, 1/20 and 1/25 of the line peak, one finds much larger red shifts: about 1.2, 1.5 and 1.8 Å, respectively. The red shift of the center of gravity of the observed line as a whole is, of course, a weighted (by intensity) mean value of the discrete shifts defined in this way, taken over the entire line profile.

We note that the red shift of the "experimental line center" is defined in the region in which the spectral line is formed by intermolecular electric fields ranging from about one Holtsmark "normal" field (at 1/2 width) to one almost twice as large (at 1/8 width). However, the red shift of the center of gravity of the line is caused by plasma microfields fluctuating over the whole range of variability. It may therefore be regarded as a quantity which corresponds approximately to the mean (macroscopic) plasma parameters—T and N_e.

Figure 1 shows, as an example, how the red shift of the center of gravity of the line depends on the wavelength range $\Delta\lambda$ (the distance from the unshifted line center) used to calculate it. This figure is drawn using the carefully reconstructed measurement data of Wiese (1972, Fig. 6) for the H_β line[2]). Near the line peaks of $H_\beta(|\Delta\lambda|<20$ Å), the well-known enhancement of the blue side over the red side is visible; away from this region a crossover occurs and the red wing is increasingly enhanced above the blue one. Thus, the red shift of the center of gravity of the whole line, after taking into

[1]) We note, however, that the H_β originates from an even upper level, but still manifests an observable red shift (cf. Wiese, 1972; Wiese *et al.*, 1972).

[2]) We reconstructed the far wings of the line ($|\Delta\lambda|\geqq 84$ Å) using the asymptotic Holtsmark distribution, $I\sim|\Delta\lambda|^{-5/2}$. They contribute exactly 5% of the global line intensity, which in the scale of Fig. 6 of Wiese is normalized to 1000 units (from the reconstruction, we obtained 1000.9 units).

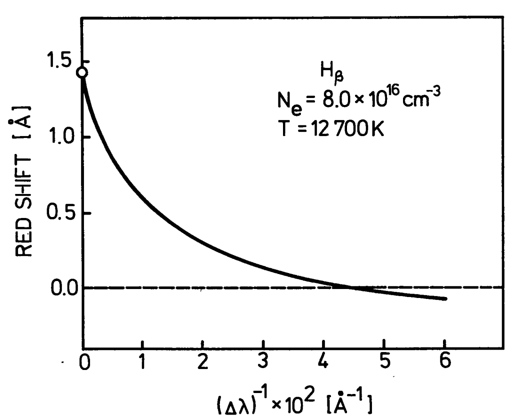

Fig. 1. The red shift of the center of gravity of the H_β line, $\int_{-\Delta\lambda}^{+\Delta\lambda} I(x)x\,dx / \int_{-\Delta\lambda}^{+\Delta\lambda} I(x)\,dx$, as a function of the half-width of the region, $\Delta\lambda$ (in both sides from the normal unshifted location of the line, $x=0$), used for calculations. Along the abscissae the reciprocals of $\Delta\lambda$ (in $Å^{-1}$) are given. The physical conditions in which the line is formed correspond to a Debye radius $D=275$ Å

account the contribution of the line wings, amounts to

$$\langle\Delta\lambda\rangle_{\text{meas}}=1.43\,\text{Å} \tag{1}$$

and is considerably larger than the shift of the experimental line center. Unfortunately, it is in the far wings that the measurement errors (relative) are largest, even in pure hydrogen plasmas where the correction for the continuum can be made exactly.

3. Attempts at a Theoretical Explanation of the Observations

We shall consider only important physical mechanisms, ignoring the well-known "trivial" causes of the hydrogen-line asymmetry such as ω^4-dependence, Boltzmann population factors, and the transformation from frequency to wavelength scale (cf. e.g. Griem, 1964). The mechanisms are the following: (1) A certain asymmetry and red shift of the center of gravity of hydrogen lines is caused by Stark-effect splitting (in a homogeneous field) when the quadratic and the higher order terms are taken into account. The intensities of the Stark components of the line also depend on the strength of the intermolecular electric field (see Nguyen Hoe *et al.*, 1965). (2) The field-ionization process due to tunnel transitions (the so-called dissolution effect), which lowers the intensity of the red components of the Balmer lines (cf. e.g. Condon and Shortley, 1963, Section 2^{17}). (3) Quadrupole and higher multipole interactions (cf. Sholin, 1969).

For the Lyman and Balmer lines formed in a plasma, effects (1) and (2) make negligible (except near to the observed series limits) contributions to the observed asymmetries and red shifts, if the far line wings are ignored in the measurements. It is not easy for these effects to appear observationally by themselves, because they contribute most at both high electron densities and for late series members where, however, the series cut-off already occurs owing to pressure effects. For the first series members, effect (1) becomes relatively important only at high densities, of the order of $N_e=10^{17}\,\text{cm}^{-3}$ or higher. For the physical conditions corresponding to the measurements of Wiese *et al.*, effect (2) is very significant only for members of the Lyman and Balmer series not earlier than Ly-δ and H_γ, respectively [see Grabowski, 1973, Eq. (22)]. When the higher series members can be observed, the relative contribution of both effects to the observed asymmetries and the red shifts appears to be still smaller. For example, Bengston and Chester (1972) have estimated that in order to produce the ratio shift/width ≈ 0.1, observed for H_{15} and H_{17} lines in low-density plasmas, an electric field $F\approx 5\times 10^4$ V/cm is required. However, at such high fields all series members beyond H_8 are already destroyed by the dissolution effect. This last effect, for the conditions pertaining to Bengston and Chester's experiment, significantly affects only lines beyond the observed series limit (cf. Grabowski, 1973).

The quadrupole and the higher multipole-interaction effects require special comment. A semi-qualitative explanation of the line asymmetry, based on the static quadrupole ion-atom interactions (arising from the inhomogeneity of the intermolecular electric field in a plasma) is given by Sholin (1969), who used the nearest neighbour approximation. Considering the first-order asymmetry term, and applying the usual perturbation calculations, he has found that in general the Stark components of all hydrogen lines crowd together on the short wavelength side and spread out on the long wavelength side of a line. Consequently, the center of gravity of the Stark-split set of line components (corresponding to a fixed ion-atom distance) experiences a quadrupole red shift given by:

$$\langle\Delta\lambda_q\rangle=\frac{\lambda_0^2}{2\pi c}\Big(\sum_k I_k\Delta\omega_k\Big)\Big/\sum_k I_k\,, \tag{2}$$

where λ_0 is the unperturbed wavelength of the line, $\Delta\omega_k$ and I_k are the frequency shift and the intensity of k-th component (using Sholin's terminology), respectively. Table 1 lists the numerical values of the red-shift $\langle\Delta\lambda_q\rangle$ of the center of gravity of the H_α and H_β lines, calculated using Sholin's (1969) dipole and quadrupole shifts and the intensity variations of all components. The results are given for several electron densities N_e. Each number in the table corresponds to a set of "line components" produced by a statistical atom under influence of the mean physical conditions [the distance of the nearest perturber is assumed to be equal to the mean ion-ion separation R_0, where conventionally $(4/3)\pi R_0^3 N_e=1$].

These quadrupole red-shifts of the "center of gravity" are smaller by the order of magnitude than the corresponding shifts of the "experimental line center" measured by Wiese *et al.* The differences between the quadrupole (Table 1) and the measured red-shifts of the center of gravity should be still larger. We note furthermore that: (1) The quadrupole red shifts decrease with the number of the line in the series, contrary to observation. (2) For the central components of the H_α and H_γ lines, the quadrupole effects give slight blue shifts (cf. Sholin), while red shifts are in fact observed (cf. Wiese). (3) Bacon (1973) took ito account both first- and second-order asymmetry terms (the quadrupole and the quadratic effect terms in the ion-atom interactions) and included electron broadening: the resulting asymmetry of Lyman-α was substantially smaller than that calculated by

Table 1. The red shifts, $\langle\Delta\lambda_q\rangle$ in Angström units, of the centers of gravity of the H_α and H_β line components (produced by a statistical atom under mean physical conditions) due to the quadrupole effect

$N_e\,[10^{16}\,\text{cm}^{-3}]$	1	5	10
H_α	0.005	0.024	0.048
H_β	0.002	0.008	0.018

Sholin. Thus, we see that the quadrupole and the higher multipole interactions are not the principal cause of the observed asymmetry of hydrogen lines, and that no theoretical explanation is available as yet for the observed red shifts of these lines [cf. conclusions of Wiese (1972) and Wiese *et al.* (1972)].

4. Red Shifts of Hydrogen Lines in the Debye Model

It is well known that in plasma conditions an ion field is screened by a "cloud" of free electrons. In the quasi-static approximation, the screening effects are taken into account fully when calculating the intermolecular ionic field-strength distribution functions in the plasma (e.g. Mozer and Baranger, 1960; Hooper, 1966, 1968). The model of plasma screening has been applied to bound atomic states near the continuum in order to estimate how pressure reduces the atomic ionization energy (Ecker and Weizel, 1958; Margenau and Lewis, 1959). However, the effect of plasma screening on the internal field and, consequently, on the intraatomic energy structure of hydrogen atoms, has until now been ignored (cf. Halenka and Grabowski, 1975). Kudrin and Tarasov (1962) have considered the Debye-screening shifts of the low energy states of one-electron systems, but their results are somewhat debatable: they claim that in one-electron ions the plasma-screening shifts are the same for all levels, and that the shifts do not appear at all for hydrogen atoms.

To reconstruct the intraatomic potential of a hydrogen-like atom inside a plasma, we used the Debye screened field, $V_D(r)$, taking into account the neutralizing action of a negatively charged background on the atomic nucleus:

$$V_D(r) = -\frac{Ze_0^2}{r}\exp(-r/D). \tag{3}$$

Here D is the Debye radius [a function of electron density N_e and of tempertaure T, $D=(kT/4\pi N_e e_0^2)^{1/2}$]; Ze_0 is the effective nuclear charge.

It can be seen from the Table 1 that for quasi-static intermolecular interactions the quadrupole effects are of secondary importance only. In this connection, note that the homogeneous ion-field model, commonly used in the Stark-profile calculations (except for the far wings) is a sufficiently good approximation. We recall that the linear Stark effect, due to quasi-static ion fields, leads to symmetrical broadening of hydrogen lines. Electron-atom collisions, within the impact theories of spectral lines broadening, also act symmetrically. We shall therefore only consider the quasi-static neutralizing effects of plasma electrons on a nucleus of charge Z (with respect to one optical electron), which appear to be responsible for the observed red shifts and asymmetries of the hydrogen and hydrogen-like spectral lines.

The Debye potential (3) is a result of time-averaging. However, it can also be regarded as a model for an instantaneous internal field in the statistical atom (averaged over space), since the broadening caused by the short-time fluctuations of a field close to the radiating atoms are taken into account by the impact approximation.

For example, the energy of the ground level and of the $2s$, $2p$ and $3s$ levels of hydrogen-like atoms, within model (3), are given by the expressions:

$$J_{1,0}(\alpha) = -R_H\left[\frac{8Za_0\alpha^3}{(2\alpha+1/D)^2} - (a_0\alpha)^2\right], \tag{4}$$

$$J_{2,0}(\alpha_0,\beta) = -\frac{R_H}{4}\left[\frac{4a_0\beta^2}{3(\beta^2-\alpha_0\beta+\alpha_0^2)}\left\{-7\beta^2+\alpha_0\beta-\alpha_0^2 + \frac{24Z}{a_0}\beta^3\frac{6\beta^2+4(2/D-\alpha_0)\beta+2\alpha_0^2-4\alpha_0/D+3/D^2}{(2\beta+1/D)^4}\right\}\right], \tag{5a}$$

$$J_{2,1}(\gamma) = -\frac{R_H}{4}\left[\frac{2Za_0(2\gamma)^5}{(2\gamma+1/D)^4} - (2a_0\gamma)^2\right], \tag{5b}$$

$$J_{3,0}(\alpha_0,\beta_0,\delta) = -\frac{R_H}{9} \cdot \left[\frac{(3a_0A)^2}{\delta^5}\{-2\delta^4+2B\delta^3-2B^2\delta^2+6BC\delta-9C^2\} + \frac{(12A)^2Za_0}{F^6} \cdot \{F^4-4BF^3+6(B+2C)F^2-48BCF+120C^2\}\right], \tag{6}$$

where

$$A^2 = \delta^7[2\delta^4-6B\delta^3+6(B^2+2C)\delta^2-30BC\delta+45C^2]^{-1},$$

$$B = -\frac{3(\beta_0+\delta)^2(\delta-\alpha_0)+(\alpha_0+\delta)^2(2\beta_0+5\alpha_0-3\delta)}{3(\alpha_0+\delta)(3\delta-5\alpha_0-2\beta_0)+3(\beta_0+\delta)(4\alpha_0+\beta_0-3\delta)},$$

$$C = \frac{(\alpha_0+\delta)^2}{12} + B\frac{(\alpha_0+\delta)}{4}, \quad F = 2\delta+1/D.$$

R_H is the Rydberg constant for the hydrogen atom, a_0—the Bohr "radius". α, β, γ and δ are the parameters whose numerical values (α_0, β_0, γ_0 and δ_0) define the variational requirements: $\partial J_{1,0}/\partial\alpha = \partial J_{2,0}/\partial\beta = \partial J_{2,1}/\partial\gamma = \partial J_{3,0}/\partial\delta = 0$, respectively. (Complete results concerning the next higher energy levels and a comment on the calculation technique will be published in another paper by Halenka.) For $D\to\infty$, our equations for the energies of low lying levels, $E_{n,l} = \min J_{n,l}$, $n=1,2,\ldots$, $l=0,1,\ldots,n-1$, reduce to the usual Bohr hydrogen-like expressions (in this limit, the expressions in the square brackets in Eqs. (4)–(6) are equal to 1).

The lower part of Fig. 2 shows as an example the fine structure of the H_α line produced by an unperturbed hydrogen atom; the upper part concerns a statistical hydrogen atom[3]) whose internal potential (3) is averaged

[3]) The atom is such that the local physical conditions in its neighbourhood are the same as the macroscopic ones.

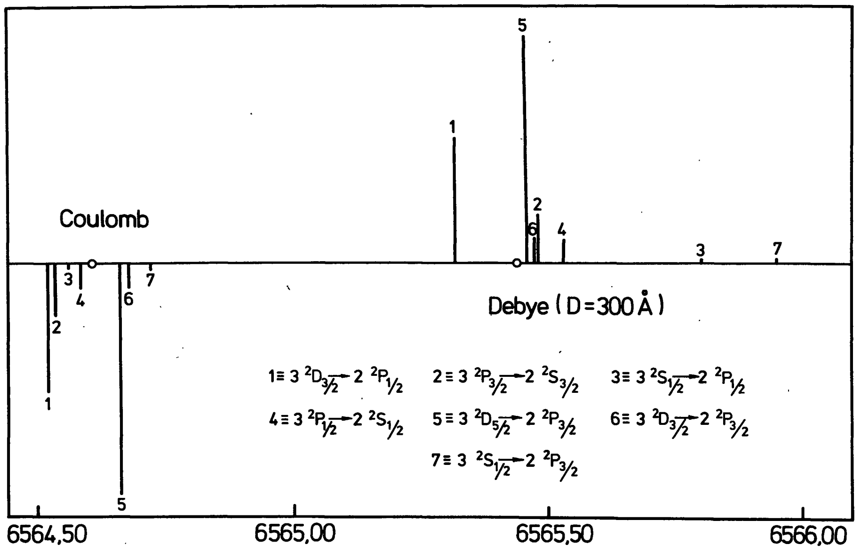

Fig. 2. The fine structure of the H_α line in the unperturbed hydrogen atom (the Coulomb field or $D=\infty$), and that in the atom with a Debye screened intraatomic field of $D=300$ Å. The red shift of the center of gravity of the set of H_α components in a plasma corresponding to the above Debye radius is equal to 0.83 Å

in a plasma of parameters T and N_e corresponding to the Debye radius $D=300$ Å[4]). The figure shows clearly the red shift due to the electron screening effect. The circles mark the positions of the centers of gravity of both sets of H_α components; the lengths of the strips represent the relative intensities of the particular components. (Corrections due to spin-orbit interactions and relativistic effects have been taken into account; in the case of the Debye calculation, the physical conditions correspond to $D=300$ Å.) In the lower part of this figure the transitions forming the H_α line are given. Note the permutations of particular components within the Debye pattern as compared to the Coulomb pattern.

Figure 3 shows as a function of the Debye radius the "screened" red shifts of the centers of gravity of the components of the first members of the Lyman, Balmer and Paschen series, produced by a hydrogen atom statistically averaged in a plasma. We emphasize that these red shifts correspond rather to the red shift of the center of gravity of a broadened line taken as a whole than to the red shifts measured by Wiese *et al.* for the so-called experimental line centers. In particular, for the physical conditions $N_e=8.0\times10^{16}\ \mathrm{cm}^{-3}$, $T=12700$ K ($D=275$ Å), we may compare the red shift of the center of gravity of the H_β line, measured [cf. Eq. (1)] and calculated as above:

$$\langle\Delta\lambda\rangle_{\mathrm{meas}}=1.43\ \text{Å}\,,\qquad \langle\Delta\lambda\rangle_{\mathrm{calcul}}=1.53\ \text{Å}\,. \tag{7}$$

The agreement between these two results is encouraging.

However, in order to compare the measured asymmetries and the red shifts of the experimental line centers with the proper theoretical quantities, one should compute the whole line profile taking into account all three broadening effects: (1) the space

[4]) In pure hydrogen plasma of atmospheric pressure this approximately corresponds to the electron density $N_e\approx6\times10^{16}\ \mathrm{cm}^{-3}$ and the temperature $T\approx12000$ K.

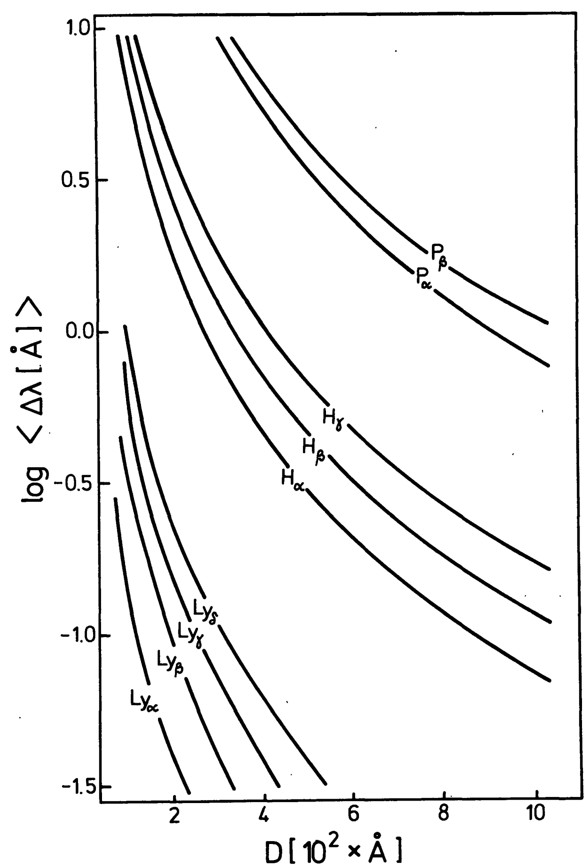

Fig. 3. The Debye-screened red-shifts of the centers of gravity of the line components of the first members of the Lyman, Balmer and Paschen series (in a logarithmic scale) as a function of the Debye radius

fluctuations of the ionic quasi-static fields, (2) the short-time electron perturbations, and (3) the red shifts due to "quasi-static" electron screening (averaged over times of the order of ion relaxations). The results of such calculations will be published later. We state simply that the splitting caused by the linear Stark effect is symmetrical and depends on the local ion concentration. The position (on a wavelength scale) of the center of symmetry of the set of Stark "line components", contributed by an atom to the observed line, coincides with the center of gravity of the proper Debye fine structure and depends on the screening effect of a local electron "cloud".

5. Conclusions

Our numerical calculations using the Debye model bring us to the following general conclusions:

(i) The lowering of the ionization energy, ΔE (caused only by the shift of the ground level towards lower bond energies), is (cf. Halenka and Grabowski, 1975) equal to[5])

$$\Delta E = \frac{Ze_0^2}{D} (CGS) . \tag{8}$$

This result is exactly the same as the depression of the ionization energy, obtained earlier by Theimer (1957, 1958) and Griem (1962) using different physical treatments of hydrogen-like systems in Debye's model.

(ii) All the components of given lines of a hydrogen-like system are shifted towards the red, but the particular components are—in principle—shifted by different amounts (cf. Fig. 2).

(iii) For fixed physical conditions, the red shift of the center of gravity of the components of a hydrogenic line increases with its order number in a series: e.g., within the Balmer series the red shift of H_β is greater than that of H_α, etc. (cf. Fig. 3). Furthermore,

(iv) the shift of a given member of a series increases for consecutive series. For example, the red shift of the Balmer H_β is greater than that of the Lyman Ly-β, etc. (cf. Fig. 3).

(v) The red shift of the center of gravity of the given line is approximately a linear function of the electron density[6]).

(vi) The red shift in an individual atom depends on the local physical conditions in the plasma. One can predict qualitatively that the statistical fluctuations of the local density of electric charges should cause asymmetry of a line, favouring the red over the blue wing.

The direction and order of magnitude of features (iii)–(vi) do agree with recent detailed measurements of hydrogen lines (cf. Points a, b, e and f in Section 2). We should like to add that, after taking into account the deformation of the internal atomic structure due to the electron-screening effects, one obtains the splitting of energy levels apparently different from that of the linear Stark effect in a homogeneous field only. One might thereby explain why the measured profiles in the line centers have a less complex structure than the calculated ones (cf. Point c).

[5]) The result appropriate for the Debye potential containing polarization term differs from our result by the numerical value of this term as by the additive constant.

[6]) This dependence can be easily exhibited for the case of the pure-hydrogen equilibrium plasma, where a unique relation exist between N_e and T.

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B. Grabowski
J. Halenka
Institute of Physics
Higher Pedagogical School
Oleska 48
PL-45-052 Opole, Poland

Addendum

During revision of the present work, a paper devoted to the quadrupole red shifts of the hydrogen lines in dense plasma was published by Demura and Sholin (1974). The red shifts of the centers of gravity of the H_α and H_β lines obtained by these authors in the nearest neighbour approximation, are larger by at least an order of magnitude than the values given in our Table 1. They are in good agreement with the red-shift measurements made by Wiese *et al.* for the so-called experimental line centers. Unfortunately, the outcomes of Demura and Sholin result from a few inaccuracies in their paper.

(1) The quadrupole red shift of the center of gravity of the line, broadened by the quasi-static interactions, is equal to

$$\langle \widetilde{\Delta\lambda}_q \rangle = \int_R \langle \Delta\lambda_q \rangle W(R) dR\,, \qquad \text{(A1)}$$

where $\langle \Delta\lambda_q \rangle$ is given by the Eq. (2), and R is the distance of the perturbing ion. From Sholin's relations $\Delta\omega_k = \Delta\omega_k(R)$ and $I_k = I_k(R)$ [Eqs. (23) and (29) in the paper of Sholin (1969)], the quantity $\langle \Delta\lambda_q \rangle$ varies as R^{-3}. Hence, the red shift of the center of gravity of the line is directly proportional to the weighted mean, $\langle R^{-3} \rangle \equiv \int_R R^{-3} W(R) dR$. In the nearest neighbour approximation for the $W(R)$ function, the quadrupole shift $\langle \widetilde{\Delta\lambda}_q \rangle$, averaged in the range $0 < R < \infty$, is equal to infinity. Cutting the range of R from below, i.e. at $R_{\min} = R_{\text{crit}}$, one obtains $\langle \widetilde{\Delta\lambda}_q \rangle \sim E_i(x)$, where $E_i(x)$ depends extremely on the cut-off $x = (R_{\text{crit}}/R_0)^3$. If, in particular, one takes $R_{\text{crit}} = a_0$ (a_0—Bohr "radius"), the expression (A1) for $\langle \widetilde{\Delta\lambda}_q \rangle$ becomes the same as the final equation of Demura and Sholin [Eq. (3) in their paper], exact to the term $+3 \sum_{k>0} I_k \Delta_k^q$ which, unfortunately, does not emerge from their formalism. Moreover, the condition $R_{\text{crit}} = a_0$ has no physical meaning because: (a) It is inconsistent with the conditions for which the starting Eq. (14) in the paper of Sholin (1969) may be applied. (b) The nearest neighbour approximation, used by Demura and Sholin to the quadrupole red-shift calculations, involves Unsöld's model for the lowering of the atomic ionization energy in the field of the nearest ion. In this (classical) model the distribution function $A(R)$ of the existence of the unionized atom is given as a step function of the atom-ion distance R:

$$A(R) = \begin{cases} 1 \text{ when } (3e_0^2/R) < |E_n|\,, \\ 0 \text{ when } (3e_0^2/R) \geqq |E_n|\,, \end{cases} \qquad \text{(A2)}$$

where E_n—the unperturbed energy of the bound state whose principal quantum number is n. Hence the minimum distance R, at which the perturbed atom may still be in n-th excitation state, is $R_{\text{crit}} \gtrsim 6a_0 n^2$. This inequality is strengthened still further by the quantum character of the field ionization (tunnel effect) as well as by the Stark-effect splitting. We note that Bacon (1973) in his calculations of the asymmetry of the Lyman-α line used $(R_{\text{crit}})_{\text{Ly}-\alpha} \approx 30a_0$, approximately the same as the value resulting from the criterion $R_{\text{crit}} \approx 6a_0 n^2$.

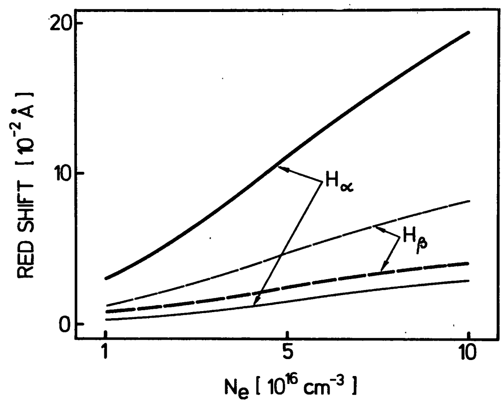

Fig. A 1. The red-shifts of the centers of gravity of the H_α and H_β line components, due to the quadrupolar interactions (thick lines) and due to the quadratic Stark effect (thin lines)

The quadrupole red shifts $\langle \widetilde{\Delta\lambda}_q \rangle$ for H_α and H_β, calculated according to (A1) in the nearest neighbour approximation in the interval of R from $R_{\text{crit}} = 6a_0 n^2$ to ∞, are shown in the Fig. A1 together with the proper shifts due to the quadratic Stark effect. It can be seen from this figure that the relative importance of the quadratic Stark effect increases, and of the quadrupole interaction decreases, with the principal quantum number of the upper level of the line.

In so far as the quadrupole red shifts are concerned, the $\langle \widetilde{\Delta\lambda}_q \rangle$ values are greater by the factor 4–6 for H_α and by the factor 2.5–4.5 for H_β with respect to the "statical" shifts given in Table 1. Nevertheless, the values of $\langle \widetilde{\Delta\lambda}_q \rangle$ are much smaller (for H_β—by an order of magnitude) than those given in the paper of Demura and Sholin (cf. their figure).

(2) Demura and Sholin have compared the red shifts (a) of the *center of gravity* (calculated quantities) with (b) the so-called *experimental line center* (measured quantities). However, these quantities are not strictly comparable (see Section 2). In fact, the discrepancy between calculated (caused by quadrupole interactions) and measured red shifts for the *centers of gravity* of lines, will be still greater than given above. In particular, under the physical conditions for which the red shifts given in the Eqs. (7) apply, the quadrupole red-shift of the center of gravity of the H_β line hardly amounts to

$$\langle \widetilde{\Delta\lambda}_q \rangle = 0.04\ \text{Å}\,. \qquad \text{(A3)}$$

In the opinion of the authors, Sholin's description, in which the quadrupole term occurs, is a limiting quasi-static approximation only. It takes into account the contribution of the nearest ion, and neglects the influence of the rest of the plasma medium. In this paper the other limit has been presented, i.e. the quasi-static mean effect of the whole of the negatively charged plasma background is taken into consideration in the Debye model. The quasi-static effect of plasma on the internal structure of hydrogenic atoms may be best described by simultaneous taking into account the following effects: (a) the resultant homogeneous field of ions mutually screened and correlated, (b) the neutralizing effect of plasma electrons on the atomic nucleus (the screening effect), and (c) the quadrupole contribution to the Stark effect caused by the field of the nearest ion, also screened by electrons.

A comparison of the results (7) and (A3), as well as the earlier discussion, shows that screening effects cannot be neglected when calculating the red shifts of hydrogen lines in dense plasmas, and that the quadrupole effects alone can in no case explain observation.

Reference

Demura, A. B., Sholin, G. W. 1974, *Opt. Spectrosc.* (USSR) **36**, 1221