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ASYMMETRY OF THE PEAKS OF THE H_β SPECTRAL LINE

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Abstract—The asymmetry parameter for the intensity peaks of the H_β line, $\delta I = (I_B - I_R)/I_B$, has been measured in a hydrogen–argon arc-plasma for electron concentrations N_e between 3×10^{22} and $10^{23}\,m^{-3}$. The observed results agree with measurements by other authors on different plasma sources. We have calculated δI, taking into account multi-ion–atom quadrupole interactions and the quadratic Stark effect. For N_e in the range from 2×10^{22} to $10^{24}\,m^{-3}$, the calculated values of δI are smaller by a factor of about 2 than the measured values. We therefore conclude that the efficiency of multi-ion–atom quadrupole interactions in the currently used statistical description [with $B(\beta)$ defined by Chandrasekhar and von Neumann] is too small.

INTRODUCTION

It is well known that H spectral lines emitted from plasmas are asymmetric and red-shifted. Various definitions, based on different parameters characterizing different parts of the line profile, have been introduced as measures of asymmetry and/or shift.[1–4] Kudrin and Sholin[1] have calculated the quantity $\delta I = (I_B - I_R)/I_B$, where I_B and I_R are the intensities of the blue and red peaks, respectively. This quantity is a measure of the asymmetry of the H_β peak intensities. The values calculated by Kudrin and Sholin are larger by a factor of nearly 2 than the experimental values determined in Refs. 5–14 from measurements on various plasma sources. The present study was undertaken with the aim of measuring values of the asymmetry parameter δI in arc plasmas for $N_e > 3 \times 10^{22}\,m^{-3}$, for which experimental data are scarce, as well as to perform a theoretical estimation of δI. Preliminary results were presented previously at the SPIG86 in Sibenik.[14]

CALCULATIONS

As shown in Ref. 1, asymmetry of the H_β peaks, results from inhomogeneities of the ion-produced electric field (ion–atom quadrupole interactions). In Ref. 1, calculations were performed in the quasi-static approximation (QSA); the electric microfield distribution (EMD), as well as the field gradient, were taken into account in the nearest-neighbour approximation (NNA). In order to obtain an improved estimate for the asymmetry parameter δI in the present paper, the NNA approach has been replaced by a multi-ion approximation and the influence of free electrons has been taken into account.

Demura and Sholin[2] showed in the Holtsmark limit that, for the reduced field strength $\beta = F/F_0$ ($F_0 \approx e_o/R_0^2$, $4/3\pi R_0^3 N_e = 1$), the components of the ion-electric field inhomogeneity tensor for interactions with any number of ions can be expressed by the universal function

$$B(\beta) = \left[3\int_0^\beta H(\beta')\,d\beta'\right] \Big/ [\beta H(\beta)] - 1 \tag{1}$$

introduced by Chandrasekhar and von Neumann.[15] To make allowances for correlations between perturbing ions, as well as for Debye screening by electrons, the $H(\beta)$ function for EMD has been replaced in the present study by the function $H_\rho(\beta)$ taken from Ref. 16 and the $B(\beta)$ function for the field gradients by the functions $B_\rho(\beta)$ calculated according to Eq. (1) using the $H_\rho(\beta)$ functions. Additional, second-order corrections connected with the homogeneous term of the ion field (quadratic Stark effect) were taken into account.

As is well known, interaction of an emitter with free electrons also leads to broadings and shifts of H spectral lines. Here, we have assumed, in agreement with the calculations of Refs. 17–19, that the shift $\Delta\omega^{e}$, caused by interaction of an emitter with free electrons, is constant within the entire range of the profile and, therefore, that δI does not depend on $\Delta\omega^{e}$. It should, however, be noted that the theoretical description of the $\Delta\omega^{e}$-shift remains unsatisfactory, as is demonstrated by measurements.[4]

Broadening is well described by the impact theory.[20] Using this description, H-line profiles I^{d} have been calculated for the approximation of ion–atom dipole interaction. Because the ion–atom quadrupole interaction and the quadratic Stark effect are second-order corrections, the H-line profile, including these corrections and also the $\Delta\omega^{e}$-shift, may be described by the expansion

$$I(\Delta\omega) \simeq I^{d}(\Delta\omega + \Delta\omega^{e}) + \Delta I(\Delta\omega). \tag{2a}$$

In the present study, we have used the impact theory to describe the $\Delta I(\Delta\omega)$ correction in simplified form, i.e., the off-diagonal matrix elements of the impact operator have been neglected. For this simplification, the ith Stark component can be treated as an isolated line with relative intensity I_i, the electron broadening of which is characterized by a Lorentzian with the impact full half-width $2\gamma_i$. Thus, the $\Delta I(\Delta\omega)$-correction to the H-line profile for the transition $n \to n'$ may be expressed as

$$\Delta I(\Delta\omega) = \left(\pi \sum_i I_i^{d}\right)^{-1} \sum_i I_i^{d}\gamma_i^{d} \int_0^{\infty} d\beta\, H_p(\beta)[(\Delta\omega - \Delta\omega_i^{d})^2 + \gamma_i^{d\,2}]^{-1}$$
$$\times \left[\frac{\Delta I_i}{I_i^{d}} + \frac{\Delta\gamma_i}{\gamma_i^{d}} + 2\frac{(\Delta\omega - \Delta\omega_i^{d})\delta\omega_i - \gamma_i^{d}\Delta\gamma_i}{(\Delta\omega - \Delta\omega_i^{d})^2 + (\gamma_i^{d})^2}\right], \tag{2b}$$

where the position of ith component is defined by

$$\Delta\omega_i = \Delta\omega_i^{d} + \delta\omega_i = (3/2)(a_0 e_0/\hbar)F_0\beta[\Delta_i^{d} + \lambda^{q}\Delta_i^{q} + \lambda^{k}\Delta_i^{k}] + \Delta\omega^{e}. \tag{2c}$$

The relative intensity is

$$I_i = I_i^{d} + \Delta I_i = I^{d}[1 + \lambda^{q}\epsilon_i^{q} + \lambda^{k}\epsilon_i^{k}], \tag{2d}$$

and the electron (half) half-width is

$$\gamma_i = \gamma_i^{d} + \Delta\gamma_i = \gamma^{d}[1 + \lambda^{q}\Gamma_i^{q} + \lambda^{k}\Gamma_i^{k}], \tag{2e}$$

where $\lambda^{q} = (1/2)(a_0/R_0)B_p(\beta)/\beta$ and $\lambda^{k} = (1/16)(a_0/R_0)^2\beta$. The superscripts d, q and k denote the dipole, quadrupole and quadratic terms, respectively. The meanings of Δ_i and ϵ_i for the dipole and quadrupole terms are the same ($\Gamma_i^{q} = 3\Delta_i^{\gamma}/2$) as in Refs. 21 and 2.† The terms introduced by the quadratic Stark effect have been calculated by using Refs. 22 and 23. They are equal to

$$\Delta_i^{k} = (2/3)\{17(n'^6 - n^6) - 3[n'^4(n_i' - n_2')^2 - n^4(n_i - n_2)^2] - 9(n'^4|m'|^2 - n^4|m|^2) + 19(n'^4 - n^4)\} \tag{2f}$$

for the position of the ith component,

$$\Gamma_i = 4[(w_i - v_i)a_{i,0} + (w_i' - v_i)a_{i,0}']/(w_i + w_i' - v_i) \tag{2g}$$

with $w_i = \{n^2[n^2 + (n_i - n_2)^2 - |m|^2 - 1]\}$ and $v_i = 4nn'(n_i - n_2)(n_i' - n_2')$ for the electron (half-width, and

$$\epsilon_i = 2\langle nn_1n_2m|\bar{d}|n'n_1'n_2'm'\rangle \sum_{p=-2}^{2} [a_{1,p}\langle n, n_1 + p, n_2m|\bar{d}|n'n_1'n_2'm'\rangle$$
$$+ a_{2,p}\langle nn_1, n_2 + p, m|\bar{d}|n'n_1'n_2'm'\rangle a_{1,p}'\langle nn_1n_2m|\bar{d}|n', n_1' + p, n_2'm'\rangle$$
$$+ a_{1,p}'\langle nn_1n_2m|\bar{d}|n'n_1', n_2' + p, m'\rangle] \tag{2h}$$

†Equation (31) in Ref. 2 has a mistake in the expression for Q_i^{γ} related to Δ_i^{γ}, it should read $Q_i^{\gamma} = 3/2$.

for the relative intensity, which is expressed by the dipole moments of the unperturbed atom and by the coefficients $a_{k,p}$ resulting from perturbation theory. These coefficients are given as follows:

$$a_{1,-2} = -2n^3[n_1(n_1-1)(n_1+|m|)(n_1+|m|-1)]^{1/2},$$

$$a_{1,-1} = 4n^3(n_1+3n_2+2|m|)[n_1(n_1+|m|)]^{1/2},$$

$$a_{1,0} = 6n^3(n_2-n_1),$$

$$a_{1,1} = -4n^3(n_1+3n_2+2|m|+4)[(n_1+1)(n_1+|m|+1)]^{1/2},$$

$$a_{1,2} = 2n^3[(n_1+1)(n_1+2)(n_1+|m|+1)(n_1+|m|+2)]^{1/2},$$

whereas $a_{2,p} = -a_{1,p}$ when n_1 is transposed with n_2.

Using the indicated approximations, numerical calculations have been performed of the asymmetry parameter δI for electron concentrations $10^{22} < N_e$, $\mathrm{m}^{-3} < 10^{24}$ and temperatures defined by values of the screening parameter equal to $\rho = 0.4$, 0.5 and 0.6 ($\rho = R_0/D$). For $\rho = 0.5$, i.e., for a typical plasma value investigated in Refs. 5–14, the values obtained for δI are fairly well approximated by

$$\delta I = 1.33 \times (N_e/10^{22}\,\mathrm{m}^{-3})^{0.32}\,\%. \tag{3}$$

The dependence of δI on temperature is negligible; when the quantity ρ varies in the range $\rho = 0.5 \pm 0.1$, relative differences in δI do not exceed 5%.

The adiabatic approximation used for the description of the $\Delta I(\Delta\omega)$ correction [Eqs. (2)] leads to overestimation of electron broadening and consequently, to corresponding underestimation of δI. On the other hand, δI is a maximum value when $\gamma_i \to 0$, i.e., the quasi-static limit. In the QSA, we have

$$\Delta I(\Delta\omega) = \left(\sum_i I_i^d\right)^{-1} \sum_i I_i \mathrm{H}_\rho(\beta)|\mathrm{d}\Delta\omega_i/\mathrm{d}\beta|^{-1}. \tag{4}$$

The resulting δI for $\rho = 0.5$ may be approximated by

$$\delta I = 1.74 \times (N_e/10^{22}\,\mathrm{m}^{-3})^{0.49}\,\%. \tag{5}$$

In this case, the temperature dependence of δI is weaker than in the approximation of Eqs. (2), whereas the contribution of the quadratic Stark effect is greater by a factor of 2.5. This contribution to δI ranges from 38% at $N_e = 10^{22}\,\mathrm{m}^{-3}$ to 77% at $N_e = 10^{24}\,\mathrm{m}^{-3}$. With increasing N_e when the multi-ion approximation of Eq. (1) is used instead of the NNA to describe the field inhomogeneity, as in the KS-theory, the contribution of the quadruple term to δI is reduced by factors of 4–6.

EXPERIMENTAL STUDIES

The plasma was produced in a wall-stabilized arc operated in Ar with a small amount of H in the central part of the arc. Atmospheric pressure was used while applying a current of 120 A. The length of the plasma column was 80 mm and the diameter 5 mm. The plasma was homogeneous along the channel axis but had radial gradients in the plasma temperature and the densities of the plasma components. The spatial resolution of our optical system allows us to select the radiation from various, nearly homogeneous plasma layers parallel to the arc axis. In this manner, the H_β line profiles from plasmas with different T and N_e could be studied. Radiation in the end-on direction was focused on the entrance slit of a grating spectrograph PGS2. The spectrum in the region of the H_β line was recorded on plates using the second-order spectrum. The spectra were traced with a microphotometer and the plate darkening was converted into an intensity scale in the usual manner. The electron density was obtained from the peak separation, as well as from the FWHM of the H_β line. The temperature of the plasma was derived from LTE, using the appropriate N_e value from line-broadening measurements and applying the total line intensity of the H_β or ArI spectral lines. The optical depth of H_β was checked by comparing the measured absolute intensity of the line peak (J_{max}) with the blackbody intensity (J_b) at the arc temperature.

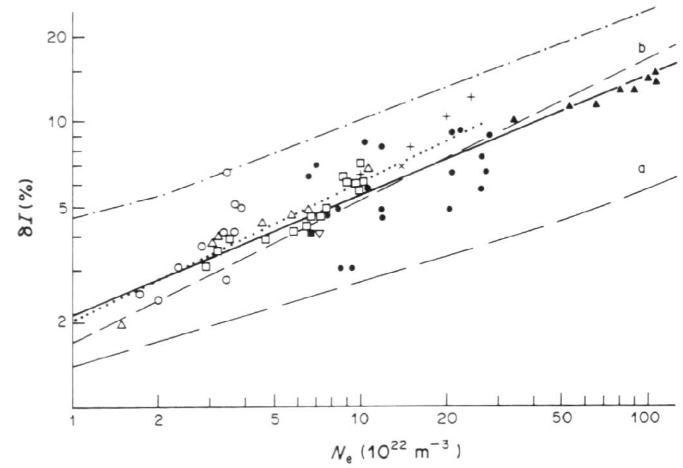

Fig. 1. The asymmetry of the H_β peaks as a function of electron density. Further explanations will be found in the text.

The ratio J_{max}/J_b was always smaller than 5%. The measured H_β line profile was corrected for optical thickness and for the so-called trivial asymmetry.[3] Next, the ratio $(I_B - I_R)/I_B$ was determined.

The ratio $\Delta\lambda_{peak\ sep.}/\Delta\lambda_{1/2}$ is very sensitive to plasma inhomogeneities. Our measurements show that, for all plasma layers studied, this ratio remains almost constant and assumes values ranging from 0.35 to 0.36.

RESULTS

The values of δI determined experimentally and calculated theoretically in the present study, as well as the results reported by other investigators, are collected in Fig. 1. The solid line represents the best fit $\delta I = (2.1 \pm 0.2) \times (N_e/10^{22}\ \mathrm{m}^{-3})^{0.41 \pm 0.03}$% for the values measured in Ref. 5 (▽), 6 (■), 7 (+), 8 (×), 9 (···), 10 (△), 11 (○), 12 (▲), 13 (●), and in this investigation (□). The numerical values for δI measured by Chotin et al[9] are represented by the dotted line in Fig. 1. The dot-dash line shows the results obtained theoretically by Kudrin and Sholin.[1] The quasi-static and nearest-neighbour approximations applied in the KS-theory yield values that are larger by about a factor of 2 that those determined experimentally. The long-dash line (a) represents results of calculations performed in the present study using Eqs. (2), whereas the short-dash line (b) represents results obtained by using Eq. (4). As was discussed earlier in connection with Eqs. (2) and (4), the expected δI from the complete impact theory should lie slightly above the line (a). Thus, the δI calculated in this paper, taking into account the quadrupole and quadratic corrections, are about 2 times smaller than the measured values. This systematic deviation may result from underestimating the quadrupole interaction for $\beta < 4$. Allowances made for screening and correlation in the $B_\rho(\beta)$ functions are rather crude, because the relation between $B(\beta)$ and $H(\beta)$ given by Eq. (1) is correct only in the Holtsmark limit.

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